simplifying idealizations. The problem mathematically formulated in [1] is restricted to a two-dimensional vertical finite plate in an infinite domain. It is noted that this idealization, as will be seen, has no exact counterpart in physical reality, and hence the corresponding solution, may it be exact or approximate, can be meaningful only when it reasonably approximates a real physical situation. The numerical calculations based on the relaxation technique in [1] must necessarily be confined in a finite domain, the extent of which was taken to be as large as practical and still consistent with the machine capability. This was done to minimize the effect of the field extent on the field behaviors in the immediate neighborhood of the plate, which was *the* region of interest in our study. That this is the case, may be readily demonstrated by the exact conduction solution as given by Professor Panton. The field extent used in our solution, $x = \pm 7{\cdot}5$ and $y = 7{\cdot}0$, corresponds approximately to ξ_0 of 3·33, with η taken to be $\pi/2$. This gives a Nusselt number of $N = \pi/\xi_0 = 0{\cdot}943$. When we increase the field extent by a factor of *four*, with $x = \pm 15{\cdot}0$ and $y = 14{\cdot}0$, we find that $\xi_0 = 4{\cdot}02$, which yields $N = 0{\cdot}782$. It is seen that the corresponding change in Nusselt number is only 17 per cent! Since the Nusselt number, being directly proportional to the temperature gradient, is a more critical measure of the temperature field, the above comparison signifies the fact that, with the field extent used in our study, the temperature profiles close to the plate are for practical purposes no longer sensitive to the size of the field. Thus, the effect of field extent on the results in [1] is not as severe as what one might expect after reading Professor Panton's discussion. Another measure to show the degree of adequacy of our chosen field can also be obtained from Professor Panton's solution, which gives values $\partial\theta^{(0)}/\partial y$ and $\partial^2\theta^{(0)}/\partial y^2$ at $x = 0$ and $y = 7{\cdot}0$ (or $\xi = \xi_0$ and $\eta = \pi/2$) of 0·04 and 0·006, respectively. These values tend to indicate that our choice of the locations of infinity was a reasonable one. Admittedly, the pure conduction solution obtained in [1] should only be interpreted as that for a vertical heated finite plate in a *large* but *finite* environment, instead of the infinite domain formulated originally. Even if the infinite-domain conduction problem could be solved, our solution in [1] would still be more meaningful on physical grounds, as will be discussed in the next paragraph. Once the nature of the approximation in our zeroth-order solution is understood, the higher-order perturbation quantities, as given in [1], which are driven by the zeroth-order temperature field, can be interpreted accordingly. It should be noted, however, that these perturbations are not valid at large distances away from the plate in view of the nature of the expansions.

Physically speaking, the case of laminar free convection along a vertical finite heated plate in an infinite field is not really very realistic. When the field extent is infinite, the inherent disturbances in the environment are likely to be amplified in the wake region, thus tending to lead to transition and turbulence. In such a situation, laminar flow only exists in the immediate neighborhood of the plate, and consequently the infinite domain formulation in [1] is not strictly applicable. On the other hand, it is possible to maintain steady laminar condition in the laboratory where the heated plate is suspended inside a large but finite-size container. In fact, all the known correlations for free convection to surfaces including vertical plates in the range of extremely small Rayleigh numbers (down to 10^{-7}) have been obtained in this way. For such a situation, it is not difficult to see that the solution given in [1] is actually *more* applicable than the one for the infinite-domain problem, even if such a solution should exist. However, it should also be pointed out that care must be exercised in making such a comparison in this extremely small Grashof-number range, in view of the fact that the size of the container may become a parameter.

In conclusion, we feel that on one hand we do agree with Professor Panton's discussion based on mathematical rigor. On the other hand, however, such mathematical rigor in difficult problems should be relaxed to permit obtaining meaningful informations in relevant physical situations. The analysis in [1] is a good example in which useful results are obtainable.

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HEAT TRANSFER AT THE INTERFACE OF DISSIMILAR METALS—THE INFLUENCE OF THERMAL STRAIN

IN HIS paper, Clausing [1] explains the phenomenon of thermal rectification between dissimilar metals on the basis of thermal distortion at the interface. In order to establish theoretical values for this effect, it is necessary to solve the relevant thermoelasticity problem for non-uniform heating at a surface. A particular solution has been obtained for the vertical displacement at the centre of a uniformly heated circular area on the surface of a semi-infinite solid [2].

Although the corresponding temperature field tends to a steady state, the displacement is not bounded with time because the solution includes the bulk expansion of the solid in which all the heat accumulates.

With a constant load system we are concerned only with the distortion of the surface due to non-uniform heating (or cooling). In the absence of radial heat loss, there would be no surface distortion if the heat input were uniformly distributed over the nominal contact area: thus, the actual distortion may be found by introducing a uniform heat source over the actual contact area (radius a) and an equal uniform heat sink over the total nominal contact area (radius b). In this case the net heat flow to the solid is zero so that the bulk expansion is excluded. If $b \gg a$, the semi-infinite solution is a reasonable approximation and it yields a steady state central displacement:

$$= \frac{Q\alpha(1 + \nu)\log(b/a)}{2\pi k}$$

where Q, the heat flow;
α, coefficient of thermal expansion;
ν, Poisson's ratio;
k, thermal conductivity.

This result provides a criterion for the direction of thermal rectification. For a given geometry and load, the thermal resistance will always be least when the externally heated body has the higher value of $\alpha(1 + \nu)/k$.

In his letter, Williams [3] suggests that thermal rectification can occur between similar metals with suitable geometry. It is impossible to reconcile this with a thermal distortion explanation since the heat flow out of one body is equal to that into the other and the surface displacements are therefore equal and opposite. The bulk elastic deformation under load is therefore unchanged by heat flow.

Furthermore, he is in error in claiming that changes of geometry can alter the direction of thermal rectification. The degree of rectification is certainly a function of the geometry but the direction depends only upon the material properties as remarked above. With either a convex or a concave contact, the areas in actual contact will contract in the heated body and expand in the cooled body. The bodies must therefore move nearer together if the contraction is greater than the expansion regardless of where on the surface the contact is initially made. This relative movement must increase the degree of conformity of the surfaces and consequently reduce the thermal resistance of the interface.

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